

Homologous Gravitational Collapse in Lagrangian Coordinate: Planetary System in Protostar and Cavity in Pre-Supernova

K.H. Tsui and C.E. Navia

*Instituto de Física - Universidade Federal Fluminense
Campus da Praia Vermelha, Av. General Milton Tavares de Souza s/n
Gragoatá, 24.210-346, Niterói, Rio de Janeiro, Brasil.*

tsui@if.uff.br

ABSTRACT

The classical problem of spherical homologous gravitational collapse with a polytropic equation of state for pressure is examined in Lagrangian fluid coordinate, where the position of each initial fluid element $\eta = r(0)$ is followed in time by the evolution function $y(t)$. In this Lagrangian description, the fluid velocity $v = dr/dt = \eta dy/dt$ is not a fluid variable, contrary to the commonly used Eulerian fluid description. As a result, the parameter space is one dimensional in η , in contrast to the (x, v) two-parameter space of Eulerian formulation. In terms of Lagrangian coordinate, the evolution function $y(t)$, which is not limited to a linear time scaling, agrees with the well established parametric form of Mestel (Mestel 1965) for cold cloud collapse. The spatial structure is described by an equation which corresponds to the one derived by Goldreich and Weber (Goldreich & Weber 1980). The continuous self-similar density distribution presents a peaked central core followed by oscillations with decreasing amplitude, somewhat reminiscent to the expansion-wave inside-out collapse of Shu (Shu 1977). This continuous solution could account for the planetary system of a protostar. There is also a disconnected density distribution, which could be relevant to cavity formation between the highly peaked central core and the external infalling envelope of a magnetar-in-a-cavity pre-supernova configuration.

Subject headings: gravitational collapse

1. Gravitational Collapse

The fundamental issue of gravitational collapse of a large but finite massive gas cloud concerns the star and nebula formation. Bonner (Bonner 1956) had demonstrated that the equilibrium of a finite isothermal gas cloud under its self-gravity could be unstable as the mass increases. Mestel (Mestel 1965) solved the collapse equation of a cold cloud with $v = dr/dt$ as the trajectory velocity, which is the Lagrangian fluid representation, to obtain the celebrated self-similar parametric solution. Lin (Lin et al. 1965) showed that an oblate cold spheroid would evolve to a disk and a prolate one would evolve to a spindle. Bodenheimer and Sweigart (Bodenheimer & Sweigart 1968) studied numerically the evolution sequence of a collapsing gas cloud with finite pressure again with $v = dr/dt$ under different initial density distributions and different surface boundary conditions. Penston (Penston 1969) analyzed analytically the Lagrangian cold collapse with a smooth maximum for density, and also put forward a self-similar analysis of an isothermal collapsing sphere in Eulerian description, where the fluid velocity v as one of the variables. Larson (Larson 1969) examined numerically through a set of conservation equations of mass, momentum, and energy in Eulerian representation the formation of a protostar, and presented in appendix C the isothermal similarity solutions. Shu (Shu 1977) studied anew the homologous collapse of an isothermal sphere with the Eulerian conservation equations. He interpreted the singular solutions, where the coefficients of the nonlinear differential equations vanish, and constructed the expansion-wave, inside-out collapse scenario. Hunter (Hunter 1977) added a new class of isothermal self-similar solutions on previously known ones. Goldreich and Weber (Goldreich & Weber 1980) examined the homologous collapse of a stellar core with a polytropic equation of state, and arrived at a differential equation that describes the radial structure. Perturbation analysis was used to study the nonspherical modes of the stellar core. Whitworth and Summers (Whitworth & Summers 1985) brought to the attention the importance of the stability of the initial isothermal gas cloud and the external driving pressure. These two factors transform each known solution into a continuum.

Recently, there have been renewed efforts on gravitational collapse. Fatuzzo (Fatuzzo et al. 2004), Lou and Wang (Lou & Wang 2006), Lou and Gao (Lou & Gao 2006) have considered self-similar solutions in Eulerian form with polytropic equation of state, and presented innovated solutions for astrophysical phenomena. Furthermore, Lou and Shen (Lou & Shen 2004) put forward the envelope-expansion (EE) and core-collapse (CC) solution for an

isothermal cloud. This EECC solution is to model outflowing stellar winds while the red giant collapses. Bian and Lou (Bian & Lou 2005) examined the shock flows of an isothermal sphere to better understand the shock structure.

We remark that, except cold gas clouds, clouds with isothermal or polytropic equation of state have been treated under Eulerian fluid description where velocity $v = v(r, t)$ is one of the variables. The time dependence of each variable is constructed as some specific power of time t . The spatial dependence is given by a set of nonlinear differential equations (Lou & Shen 2004). The singular solutions of this system define the singular sonic surfaces, which separate subsonic and supersonic regions in the (x, v) two-parameter space. Along the sonic surfaces, the flow is subsonic/supersonic relative to the similarity x profile. Collapse solutions cross from subsonic to supersonic regions along the sonic lines. Sometimes shock waves are needed to bridge the crossing to meet the boundary conditions. Aimed to describe the collapse, representing the time dependence with different powers of a linear time scaling for different variables would certainly fail to describe the time history of collapse. We could only expect such self-similar solutions with a linear time scaling be valid during a specific time span and over a specific radial interval. Here, we use the alternative Lagrangian description, where the fluid velocity is not a variable, to construct the self-similar collapse having a polytropic equation of state. The evolution function is solved consistently together with the spatial structures.

2. Lagrangian Similarity

Gravitational collapse of a spherical cloud is described by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 , \quad (1)$$

$$\rho \left\{ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right\} = -\nabla p - \rho \frac{GM_*}{r^2} \hat{r} - \rho \frac{GM(r, t)}{r^2} \hat{r} , \quad (2)$$

$$\frac{\partial}{\partial t} \left(\frac{p}{\rho^\gamma} \right) + (\vec{v} \cdot \nabla) \left(\frac{p}{\rho^\gamma} \right) = 0 . \quad (3)$$

Here, ρ is the mass density, \vec{v} is the gas cloud velocity, p is the gas pressure, γ is the polytropic index, M_* is a point mass at the center, if any, and $M(r, t)$ is the gas mass within a sphere of radius r at time t where

$$M(r, t) = \int_0^r 4\pi r'^2 \rho(r', t) dr' . \quad (4)$$

We note that Eq. 1 can be written alternatively as

$$\frac{\partial M}{\partial t} + v \frac{\partial M}{\partial r} = C(t) = 0 , \quad (5)$$

where $C(t)$ is a constant independent of r and varies only in time. By taking $C(t) = 0$, we would have an inhomogeneous cloud that is stationary in time. For a spherical collapse with $\vec{v} = v\hat{r}$, we use Lagrangian fluid representation with $r(t) = r(0)y(t) = \eta y(t)$, where the Lagrangian label η is the initial position of the fluid element, and $y(t)$ is the dimensionless evolution function with $y(0) = 1$ that describes the time history of the fluid element. The velocity can then be written as

$$v = \eta \frac{dy}{dt} . \quad (6)$$

Contrary to the Eulerian fluid description, the fluid velocity v here is not a variable which substantially simplifies the analysis.

We now transform the Eulerian independent variables $(r(t), t)$ to Lagrangian independent variables $(\eta, y(t))$. Furthermore, we note that the total time derivative in Eulerian representation $(\partial/\partial t + v\partial/\partial r)$ corresponds to the partial time derivative in Lagrangian representation $\partial/\partial t$. We now write the variables in separable form of y and η , and try to determine the evolution function $y(t)$ and the spatial configuration in η under similarity. Following Tsui and Serbeto (Tsui & Serbeto 2007), we have from Eq. 5 and Eq. 3

$$M(\eta, y) = M_0 \frac{1}{y^0} \bar{M}(\eta) , \quad (7)$$

$$F = \frac{p}{\rho^\gamma} = \frac{p_0}{\rho_0^\gamma} \frac{1}{y^0} \bar{F}(\eta) , \quad (8)$$

where M_0 carries the physical dimension of $M(\eta, y)$ such that $\bar{M}(\eta)$ is dimensionless, and likewise are p_0/ρ_0^γ and $\bar{F}(\eta)$. We note that F is only a function of entropy which remains constant in time and in space. For this reason, we have $\bar{F}(\eta) = 1$. Besides, from Eq. 4 and with $\rho(\eta, y) = \rho_0 Y(y) \bar{\rho}(\eta)$, we have

$$M(r, t) = y^3 \int_0^\eta 4\pi \eta'^2 \rho_0 Y(y) \bar{\rho}(\eta') d\eta' . \quad (9)$$

Comparing with Eq. 7 gives

$$\rho(\eta, y) = \rho_0 \frac{1}{y^3} \bar{\rho}(\eta) , \quad (10)$$

$$\bar{M}(z) = \int_0^z 3z'^2 \bar{\rho}(z') dz' , \quad (11)$$

$$M_0 = \frac{4\pi}{3} \frac{\rho_0}{a^3} , \quad (12)$$

where we have introduced the scaling a to write η in normalized form $z = a\eta$. Making use of Eq. 8, we now have

$$p(\eta, y) = p_0 \frac{1}{y^{3\gamma}} \bar{\rho}^\gamma(\eta) . \quad (13)$$

As for Eq. 2, in terms of Lagrangian coordinate and with $\gamma = 4/3$, it reads

$$y^2 \frac{d^2 y}{dt^2} = -\frac{p_0 a^2}{\rho_0} \frac{1}{\bar{\rho}} \frac{1}{z} \frac{\partial \bar{\rho}}{\partial z} - \frac{GM_* a^3}{z^3} - \frac{GM_0 a^3}{z^3} \bar{M} = C . \quad (14)$$

This equation has the temporal and spatial parts separated with C as the separation constant.

3. Evolution Function

Choosing $C = -NGM_0a^3$ in Eq. 14 and defining $\tau = (GM_0a^3)^{1/2}t = \omega t$, the temporal part of Eq. 14 is

$$\frac{d^2y}{d\tau^2} = -\frac{N}{y^2} . \quad (15)$$

Taking $N = 0$ would give $dy/d\tau = -n$ and $y(\tau) = y(0) - n\tau$, which reproduces the linear time scaling of the Eulerian formulation. For $n = 0$, we have the equilibrium state of a gravitating spherical cloud. For $N > 0$, $y(\tau)$ has a downward curvature. This is compatible to an inflowing fluid to the center with $y(\tau)$ decreasing from its initial value of $y(0) = 1$. The first integral of this equation is

$$\left(\frac{dy}{d\tau}\right)^2 + \left(-\frac{2N}{y}\right) = -H . \quad (16)$$

The terms on the left side could be interpreted as the kinetic and potential energies and the constant $-H$ on the right side could be regarded as the total energy. The inflow rate is given by

$$\frac{dy}{d\tau} = \pm \left[\left(\frac{2N}{y} - H \right) \right]^{1/2} = - \left[\left(\frac{2N}{y} - H \right) \right]^{1/2} . \quad (17)$$

With $H = 2N$, the initial velocity $dy/dt = 0$ starts from zero, and Fig.1 shows the evolution function $y(\tau)$ and its time derivative $dy/d\tau$ respectively.

We remark that this evolution function actually corresponds to the parametric time function of Mestel (Mestel 1965). Taking $y = \cos^2 \theta$, Eq. 16 with $2N = H$ gives

$$\left(\frac{dy}{d\tau}\right)^2 = H \tan^2 \theta .$$

With $dy/d\tau = 2 \cos \theta \sin \theta d\theta/d\tau$, we have

$$2 \cos^2 \theta d\theta = (1 + \cos 2\theta) d\theta = H^{1/2} d\tau .$$

Integrating once then gives

$$\theta + \frac{1}{2} \sin 2\theta = H^{1/2} \tau , \quad (18)$$

which is the well established parametric solution of Mestel.

4. Spatial Structure

With $C = -NGM_0a^3$, the spatial part of Eq. 14 is

$$\frac{p_0a^2}{\rho_0} \frac{1}{GM_0a^3} \frac{1}{\bar{\rho}} \frac{1}{z} \frac{\partial \bar{p}}{\partial z} + \frac{\bar{M}}{z^3} = N . \quad (19)$$

With $\bar{p} = \bar{\rho}^\gamma$, $C_{s0}^2 = \gamma p_0/\rho_0$, and $v_{ff}^2 = 2GM_0a$ where ff denotes free fall, we get

$$\frac{C_{s0}^2}{v_{ff}^2} \frac{2}{(\gamma - 1)} z^2 \frac{\partial}{\partial z} \bar{\rho}^{\gamma-1} = - \int_0^z \bar{\rho}(z') 3z'^2 dz' + Nz^3 . \quad (20)$$

Denoting $\alpha = C_{s0}^2/v_{ff}^2$, we get

$$2\alpha z^2 \frac{\partial}{\partial z} \bar{\rho}^{\gamma-1} = -3(\gamma - 1) \int_0^z (\bar{\rho}(z') - N) z'^2 dz' . \quad (21)$$

With $\gamma = 4/3$, and writing $\bar{\rho}^{1/3} = q$ and $\bar{\rho} = q^3$, Eq. 21 becomes

$$2\alpha z^2 \frac{\partial q}{\partial z} = - \int_0^z (q^3(z') - N) z'^2 dz' . \quad (22)$$

Differentiating this equation once leads to the following equation, which is similar to Eq.6 of Goldreich and Weber (Goldreich & Weber 1980),

$$2\alpha \frac{1}{z^2} \frac{\partial}{\partial z} \left(z^2 \frac{\partial q}{\partial z} \right) + q^3 = N . \quad (23)$$

In Goldreich and Weber, N is their free parameter λ which approaches zero, and $q(0)$ is chosen as unity. In our case, α and $q(0)$ are the free parameters. We remark that Bodenheimer and Sweigart (Bodenheimer & Sweigart 1968) had pointed out the importance of the parameter α . The most important difference is our constant $N = 1$, which comes from the separation constant C . Should we take $C = 0$ and thus $N = 0$, we would have $dy/d\tau = -n$ and $y = y(0) - n\tau$ from Eq. 15. This would recover the linear time scaling of the Eulerian similarity and Eq. 23 would equal to Eq.6 of Goldreich and Weber. The fact that $N = 1$ not only provides an evolution function of Mestel, but also changes qualitatively the nature of spatial solutions.

With $\alpha = 0$ for a cold fluid, the equilibrium requires $dy/d\tau = 0$ with $y(\tau) = y(0) = 1$. This means $N = 0$ and $n = 0$, and it is obvious from Eq. 22 that such a cold fluid equilibrium does not exist, other than $q(z) = 0$. As for the collapse solution of $N > 0$, we have the uniform density sphere solution of

$$\bar{\rho}(z) = q^3(z) = N .$$

This states that a uniform density cold sphere under homologous infall stays as a uniform sphere. By taking $N = 1$, ρ_0 amounts to the density of the sphere. Making use of Eq. 11, $\bar{M}(z_{max}) = z_{max}^3$, and therefore $M_0 z_{max}^3$ is the mass of the entire uniform sphere of z_{max} .

With $\alpha \neq 0$, equilibrium configuration with $N = 0$ gives a monotonically decreasing profile that ends at a finite $z_{max} < \infty$ with $q(z_{max}) = 0$. With $q(0) = 2$ and $\alpha = 100$, the profiles of $\bar{\rho} = q^3$ and q are shown in Fig.2 with $z_{max} = 40$. As for the collapse solution, we note that when $q > N = 1$, the integral of Eq. 22 is positive, and $\partial q / \partial z$ is negative with q decreasing. As the upper limit z of the integral increases, q would decrease to less than unity, and the integral would begin to decrease making $\partial q / \partial z$ to turn around. We would then get oscillating solutions about $q = 1$, as shown in Fig.3. This oscillating structure with a progressively smaller amplitude shows that the central core collapses faster than the external oscillating envelope. This spatial structure bears great resemblance to the expansion-wave inside-out collapse scenario of Shu (Shu 1977). For a finite system with $z_{max} < \infty$, the solution is a function of $q(0)$ for a given α . As $q(0)$ increases with the same α , the first minimum becomes more extensive, and followed by damped ripples about $q = N = 1$. The peak about $q(0)$ corresponds to the central core, and the oscillating part amounts to the external envelope.

This spherical oscillating envelope structure of $q(z)$ could be very relevant to protostar planetary system. The locations of the gaseous planets would be given by the oscillating peaks of $\bar{\rho} = q^3$. In the presence of rotation, as collapse proceeds, the spherical cloud would flatten to the equatorial plane, and the high density shells in the spherical cloud would become dense rings in the equatorial disk forming gaseous planets. Rotation would also stabilize the planets on their orbits as the central pre-protostar continues to collapse. Since the peaks are decreasing in amplitude, the gaseous planets would have a decreasing mass as distance increases. This happens to be the case of the giant planets of our Solar system, with the exception of Uranus which has its rotational axis on the ecliptic plane indicating a possible collision with substantial mass loss.

The fact that the boundary condition at z_{max} for equilibrium with $q(z_{max}) = 0$ differs from that for collapse with $q(z_{max}) \approx 1$ should not cause concern. Any self-similar solution, either Eulerian or Lagrangian, relies on writing the temporal and spatial dependences of any dynamic variable in separable form. This excludes the initial period of evolution of the real system, which is not amenable to similarity treatment. As a result, the self-similar initial

time $\tau = 0$ is not the initial time $t = 0$ of the real collapsing system. Thus, the homologous boundary condition at $\tau = 0$ need not be the boundary condition of equilibrium.

For $q(0) \gg 1$ and with a smaller α , the first minimum plunges to negative q as in Fig.4, and becomes positive again at $z_0 = 4.5$. As a result, the external part becomes disconnected physically from the central core, and a cavity is formed between the central core and the external envelope. The envelope will collapse under the gravitational field of the core plus the self-gravity. Such a configuration is most relevant to pre-supernova (magnetar-in-a-cavity model (Uzdensky & MacFadyen 2007)). The envelope of this disconnected system is described by

$$2\alpha z^2 \frac{\partial q}{\partial z} = -(\gamma - 1) \left(\frac{M_*}{M_0} + \bar{M} - 1 \right) , \quad (24)$$

$$\bar{M}(z) = \int_{z_0}^z q^3(z') 3z'^2 dz' ,$$

where the integral $\bar{M}(z)$ starts at a disconnected $z_0 > 0$. With $\gamma = 4/3$ and neglecting the self-gravity $\bar{M}(z)$, we have the monotonic envelope profile of

$$6\alpha q(z) = 6\alpha q(z_0) - \left(\frac{M_*}{M_0} - 1 \right) \left(\frac{1}{z_0} - \frac{1}{z} \right) . \quad (25)$$

5. Model Parameters and Sonic Surfaces

This model has two parameters M_0 and a to normalize time t and radial coordinate η , that need to be determined. To obtain these two important parameters, we begin with observations of a collapsed configuration, such as Swift (Swift et al. 2005). Observing the central core dimension, we denote it by r_1 which separates from the first minimum. This position is given by

$$r_1(\tau_{end}) = y(\tau_{end})\eta_1 = y(\tau_{end})r_1(0) , \quad (26)$$

where τ_{end} is the end time of the evolution function with $y(\tau_{end})$ vanishingly small, but nonzero. This $y(\tau_{end})$ has to be estimated independently. From the peak core density and the asymptotic envelope density, we can get the normalized $q(0)$ from $\bar{\rho}(0) = q^3(0)$. With $q(0)$ determined by observations, we get the analytic solution of $\bar{\rho}(z)$ with a given $\alpha = C_{s0}^2/v_{ff}^2$. We choose $\alpha = \alpha_1$ such that the inflection point $z_1 = a\eta_1 = 1$ to get

$$1 = z_1 = a\eta_1 = ar_1(0) = \frac{y(0)}{y(\tau_{end})}r_1(\tau_{end}) . \quad (27)$$

With $y(0) = 1$ and $y(\tau_{end})$ estimated, we can construct backwards in time the position r_1 at $\tau = 0$ giving

$$a = \frac{1}{\eta_1} . \quad (28)$$

If the collapsed configuration allows the identification of the number of oscillations on the envelope, we can compare with the analytic $\bar{\rho}(z)$ to locate z_{max} . By estimating the total mass M of the collapsed configuration, we then have

$$M = M_0 z_{max}^3 , \quad (29)$$

which gives M_0 as the mass in unit volume of $z^3 = 1$. With M_0 and a determined, we go back to the parameter $\alpha = \alpha_1$ to get $C_{s0}^2 = \alpha_1 G M_0 a$. We now calculate

$$\bar{M}(z_{max}) = \int_{z_0}^{z_{max}} \bar{\rho}(z') 3z'^2 dz' = z_{max}^3 . \quad (30)$$

Since $\bar{\rho}(z)$ oscillates about unity, $\bar{M}(z)$ needs to be integrated in z . However, $\bar{M}(z_{max})$ can be evaluated easily as above, since the collapse is merely a redistribution of densities of an initially uniform sphere with $\bar{\rho}(z) = 1$, thus giving the second equality of the above equation.

In the Eulerian formulation, the singular sonic surfaces divide the (x, v) two-parameter space in subsonic and supersonic regions, where the flow is subsonic/supersonic relative to the similarity profile (Whitworth & Summers 1985). Complete collapse solutions have to cross these regions on the sonic lines. In our Lagrangian formulation, the fluid velocity, which is not a variable, and the sound speed are

$$v = \eta \frac{dy}{dt} = (GM_0 a)^{1/2} z \frac{dy}{d\tau} . \quad (31)$$

$$C_s = \left(\frac{\gamma p}{\rho} \right)^{1/2} = \left(\frac{\gamma p_0}{\rho_0} \right)^{1/2} q^{1/2}(z) = C_{s0} q^{1/2}(z) = C_{s0} \bar{C}_s(z) . \quad (32)$$

We define the time dependent sonic function

$$S(z, \tau) = \frac{v}{C_s} = \frac{1}{(2\alpha)^{1/2}} \frac{z}{\bar{C}_s(z)} \frac{dy}{d\tau} = \bar{S}(z) \frac{dy}{d\tau} . \quad (33)$$

With $q(0) = 5$ and $\alpha = 5$, the spatial part $\bar{S}(z)$ is shown in Fig.5 with oscillations in z . At $\tau = 0$ with $dy/d\tau = 0$, the time dependent sonic function $S(z, \tau) = 0$ everywhere. As $\tau > 0$ and $dy/d\tau$ increases, $S(z, \tau)$ rises up and will cross the horizontal line of unit value at some point z_s where $S(z_s, \tau) = 1$. This is the time dependent sonic surface $z_s(\tau)$. As τ increases, this sonic surface moves relative to the similarity profile towards the center. Since $dy/d\tau$ is related to the fluid velocity $v = \eta dy/d\tau$, the movement of $z_s(\tau)$ amounts to the fluid velocity or flow speed with respect to the similarity profile (Whitworth & Summers 1985). We, therefore, have a picture of sonic surface which is compatible to that of the Eulerian formulation (Lou & Wang 2006). Nevertheless, in the Lagrangian formulation, v is directly incorporated in the evolution function, and the spatial structures are described by Eq. 22, which is analytic everywhere in the one dimensional parameter space η . There is no need of sonic lines to cross subsonic/supersonic regions.

6. Self-Organization and Conclusions

Contrary to laboratory self-organized phenomena (Hasegawa 1985), astrophysical events given enough time with specific patterns of outcome are self-organized, by definition. Gravitational collapses, extragalactic jets (Tsui & Serbeto 2007), planetary nebulae (Tsui 2008), and supernovae are some of the examples. If, and only if, self-similar description is able to represent the astrophysical configurations in late times, it could only mean that these self-similar solutions are the attractors of dynamic time evolutions from a large set of random initial conditions. For the specific case of gravitational collapse, the traditional Eulerian homologous formulation, where the fluid velocity v is one of the variables, has the time dependence of each variable expressed in a given power of a linear time scaling. The spatial dependence is described by a set of nonlinear differential equations. The sonic surfaces and sonic lines are crucial in constructing complete solutions in the (x, v) two-parameter space.

We have presented a Lagrangian homologous formulation, where the fluid velocity v is not a variable, but is derived from the evolution function $y(t)$. The fact that v is not a variable is extremely important, because the parameter space is now reduced to one dimensional in η . In particular, no sonic lines are needed to cross subsonic/supersonic regions. The crossing is embedded in the spatial structures. The time dependence of each variable is expressed in a given power of the evolution function, which is solved consistent to similarity. This evolution function $y(t)$ agrees with the well established parametric solution of Mestel (Mestel 1965). The spatial structure in the Lagrangian coordinate η is described by an equation similar to that derived by Goldreich and Weber (Goldreich & Weber 1980) with solutions covering the central core and the external envelope. In particular, the spatial solutions generate configurations that are relevant to planetary system in protostar and cavity in pre-supernova. The most important points of this Lagrangian description are that the evolution function is not limited to a linear time scaling, and the spatial solutions appear to be compatible to physical systems. These factors greatly enhance the scope of homologous treatment. We have the believe that the Lagrangian homologous description might be the mathematical tool to describe some self-organized astrophysical phenomena.

REFERENCES

- Bian, F.Y. & Lou, Y.Q., 2005, MNRAS, 363, 1315-1328.
- Bodenheimer, P. & Sweigart, A., 1968, ApJ, 152, 515-522.
- Bonner, W.B., 1956, MNRAS, 116, 351-359.
- Fatuzzo, M., Adams, F.C., & Myers, P.C., 2004, ApJ, 615, 813-831.
- Goldreich, P. & Weber, S.V., 1980, ApJ, 238, 991-997.
- Hasegawa, A., 1985, Advances in Physics 34, 1-42.
- Hunter, C., 1977, ApJ, 218, 834-845.
- Larson, R.B., 1969, MNRAS, 145, 271-295.
- Lin, C.C., Mestel, L., & Shu, F.H., 1965, ApJ, 142, 1431-1446.
- Lou, Y.Q., & Shen, Y., 2004, MNRAS, 348, 717-734.
- Lou, Y.Q., & Gao, Y., 2006, MNRAS, 373, 1610-1618.
- Lou, Y.Q., & Wang, W.G., 2006, MNRAS, 372, 885-900.
- Mestel, L., 1965, MNRAS, 6, 161-198.
- Penston, M.V., 1969, MNRAS, 144, 425-448.
- Shu, F.H., 1977, ApJ, 214, 488-497.
- Swift, J.J., Welch, W.J., & Francesco, J.D., 2005, ApJ, 620, 823-834.
- Tsui, K.H. & Serbeto, A., 2007, ApJ, **658**, 794-803.
- Tsui, K.H., 2008, A&A, **491**, 671-680.
- Uzdensky, D.A. & MacFadyen, A.I., 2007, ApJ, 669, 546-560.
- Whitworth, A. & Summers, D., 1985, MNRAS, 214, 1-25.

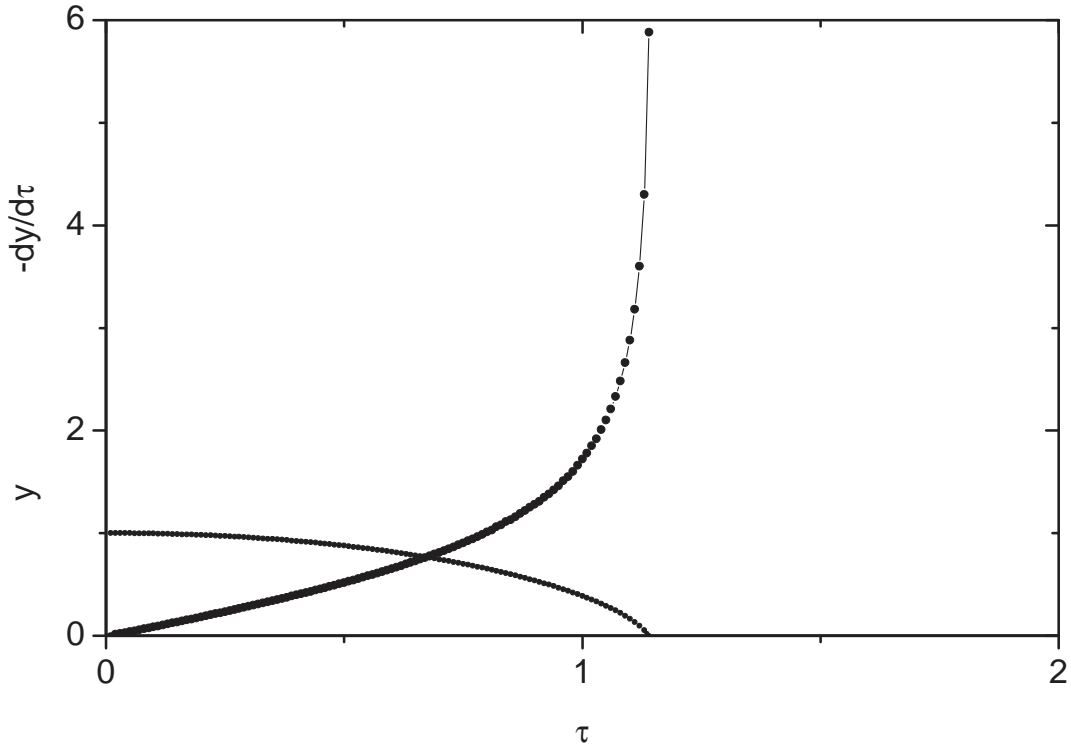


Fig. 1.— The evolution function y in thin line and its negative time derivative $-dy/d\tau$ in thick line are plotted as a function of the normalized time τ .

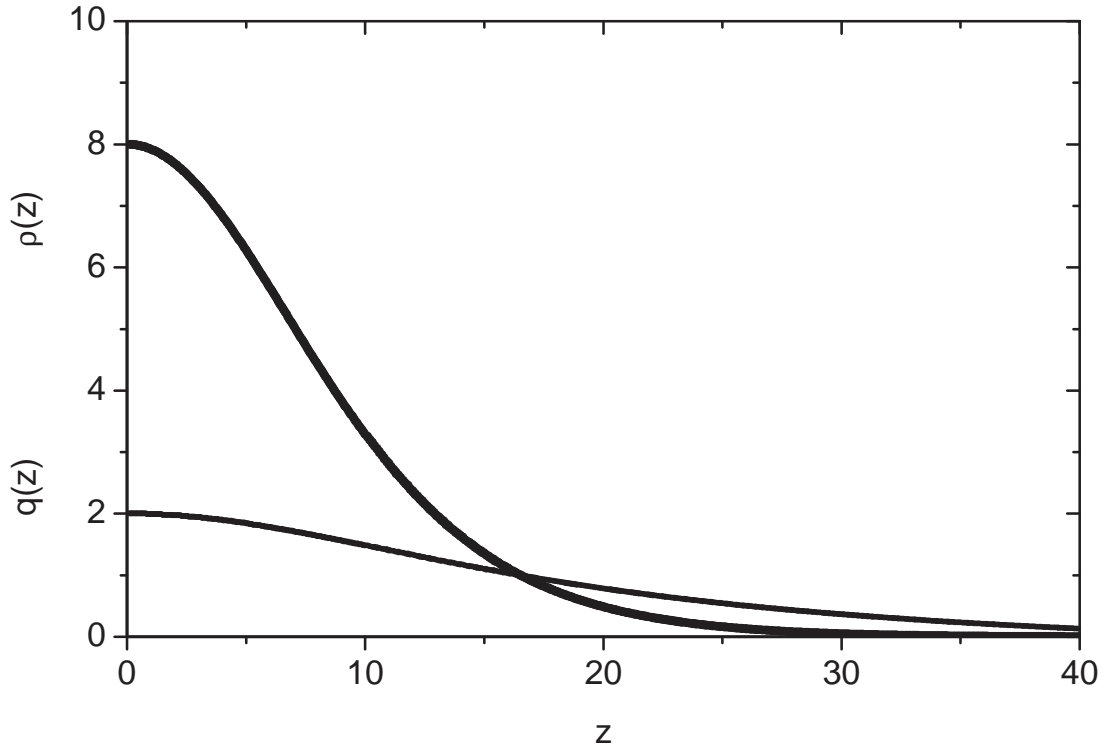


Fig. 2.— The equilibrium profile of density $\bar{\rho} = q^3$ in thick line and q in thin line are shown as a function of the normalized distance z with $q(0) = 2$ and $\alpha = 100$ giving $z_{max} = 40$.

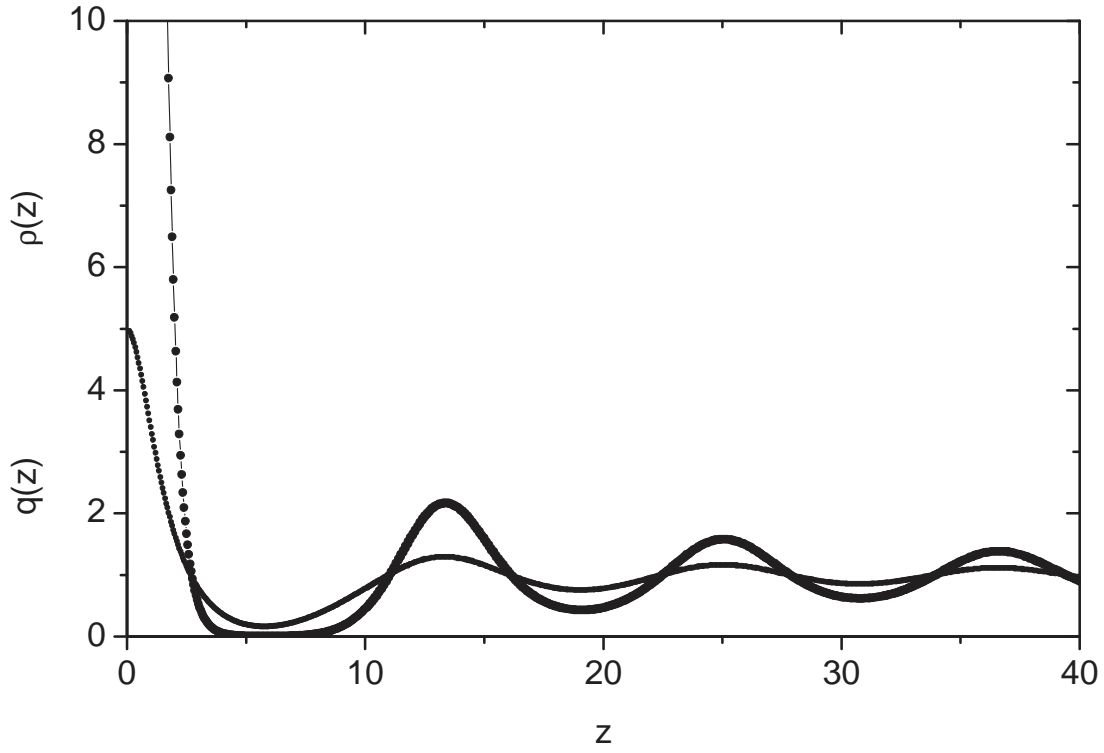


Fig. 3.— The collapsing profile of density $\bar{\rho} = q^3$ in thick line and q in thin line are shown as a function of the normalized distance z with $q(0) = 5$ and $\alpha = 5$. The peak density around $z = 0$ is off scale.

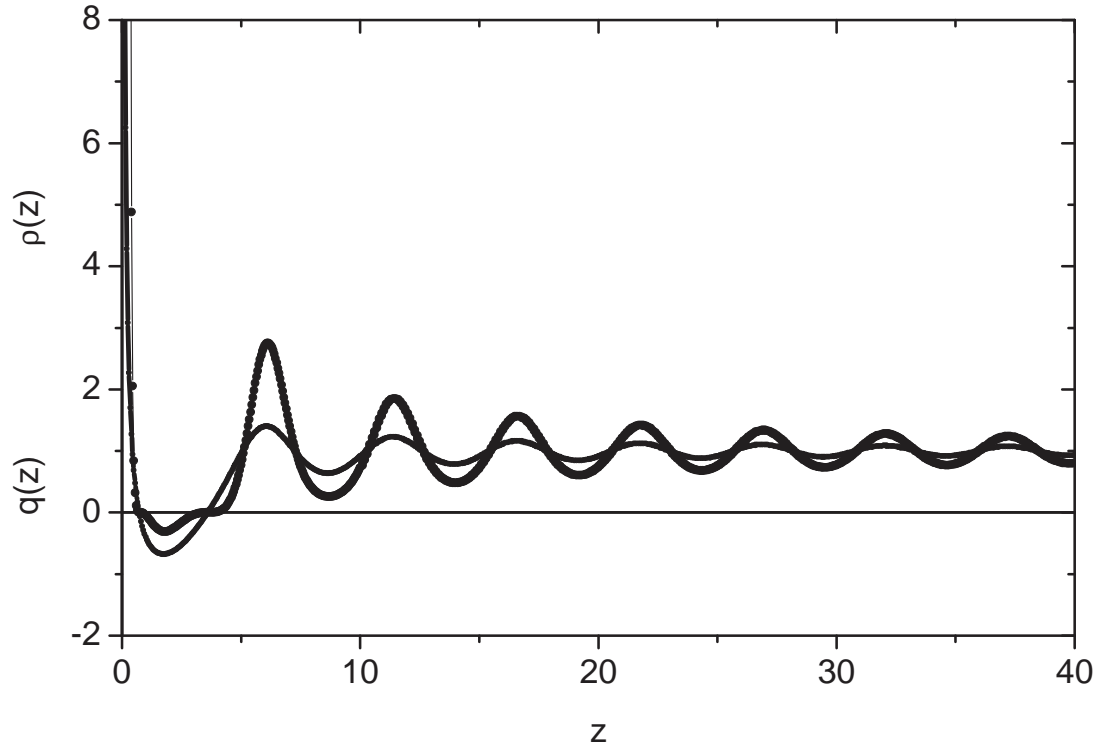


Fig. 4.— The collapsing profile of density $\bar{\rho} = q^3$ in thick line and q in thin line are shown as a function of the normalized distance z with $q(0) = 20$ and $\alpha = 1$ giving $z_0 = 4.5$. The peak $\bar{\rho}$ and q around $z = 0$ are off scale.

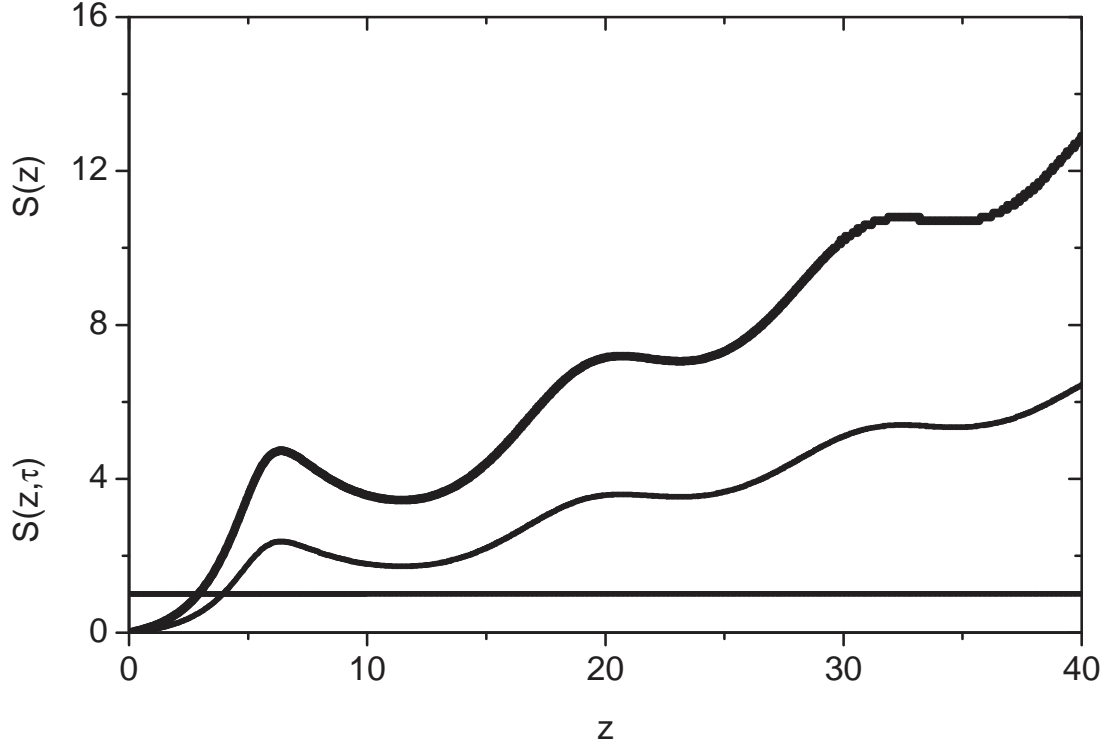


Fig. 5.— The spatial part of the sonic function $\bar{S}(z)$ in thick line and the time dependent sonic function $S(z, \tau)$ in thin line at the moment of $dy/d\tau = 0.5$ are plotted as a function of the normalized distance z with $q(0) = 5$ and $\alpha = 5$. The sonic surface is at $z_s = 3.95$.